Experiment 7

LINEAR MOMENTUM

In this experiment you will be introduced to the definition of linear momentum. You will learn the
difference between elastic and an inelastic collision. You will explore how to determine the amount of
momentum before and after elastic and an inelastic collision. In order to study these effects you will use a
motion sensor. For the collision you will use the frictionless track and carts equipped with the necessary
attachments to undergo elastic and inelastic collisions.

Objectives
1. Measure the velocities of the carts before and after the collisions
2. Calculate the momentum and the kinetic energies of the cart before and after the collisions
3. In the case of inelastic collisions, you will verify the law of conservation of momentum, viz. compare
   the momentum before and after the collision for each cart based on the measured masses and velocities
4. In the case of elastic collisions, you will verify the law of conservation of momentum, viz. compare the
   momentum before and after the collision for each cart based on the measured masses and velocities.
   You will also study the effect of such a collision on the kinetic energy of each cart.

Hypothesis
How does the momentum before elastic and an inelastic collision compare to the momentum after the
collision?

If two objects with equal but opposite momenta collide head-on non-elastically, what is their shared
velocity after the collision?

Theory
If a mass \( m \) moves along a straight line with a velocity \( v \), the linear momentum of the mass is defined as \( p = mv \). Momentum is a vector quantity and thus has a magnitude and a direction.

Collisions:
Figure 7-1 shows two carts with mass \( m_1 \) and \( m_2 \) and with initial velocities \( v_{1i} \) and \( v_{2i} \), respectively
headed for a collision (\( v_{1i} > v_{2i} \)) in one dimension (i.e. the motions before and after the collision are along a
single axis). The two carts form the system under consideration that is isolated from other effects. We
write the law of conservation of linear momentum for this two-body system as the following: Total
momentum before the collision = Total momentum after the collision

\[
m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}
\]

Where \( v_{1f} \) and \( v_{2f} \) are the final velocities of the carts after the collision. If the initial velocities of the carts
are known and one final velocity is known then using the equation above we can calculate the other final
velocity.

![Figure 7-1 Two carts have a collision while traveling at constant speeds](image)
Completely inelastic collision

In this case the two carts stick together after the collision as seen in Figure 7-2. We let the second cart, with mass \( m_2 \) stay at rest, \( v_{2i} = 0 \) while the first cart moves with an initial velocity \( v_{1i} \). After the collision the two masses stick together and move with a common velocity \( V \). Conservation of momentum requires that:

\[
m_1 v_{1i} = (m_1 + m_2)V
\]

\[
V = \frac{m_1 v_{1i}}{m_1 + m_2} \tag{7-2}
\]

In the equation above, if we know the masses and the initial velocity, we can calculate the velocity of the masses after the collision. As expected, this velocity will be less than the initial velocity of the first mass since \( \frac{m_1}{m_1 + m_2} \) must be less than unity. In inelastic collisions the linear momentum is conserved.

Elastic collisions:

Figure 7-3 shows two carts with mass \( m_1 \) and \( m_2 \) and with initial velocities \( v_{1i} \) and \( v_{2i} \) respectively headed for a collision in one dimension (i.e. the motions before and after the collision are along a single axis). The two carts form the system under consideration that is isolated from other effects. We write the law of conservation of linear momentum for this two-body system as the following:

\[
m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f} \tag{7-3}
\]

We write the law of conservation of energy for this two-body system as the following:

\[
\frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2 \tag{7-4}
\]

Where \( v_{1f} \) and \( v_{2f} \) are the final velocities of the carts after the collision. Equations 7-3 and 7-4 can be solved simultaneously to give equations 7-5 and 7-6 for the velocities after the collision in terms of the velocities before the collision.

\[
v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} + \frac{2m_2}{m_1 + m_2} v_{2i} \tag{7-5}
\]
\[ v_{2f} = \frac{2m_1}{m_1 + m_2} v_i + \frac{m_2 - m_1}{m_1 + m_2} v_{2i} \]

7-6

**Before**

\[ m_1 \quad v_{1i} \quad v_{2i} \]

**After**

\[ m_1 \quad v_{1f} \quad m_2 \quad v_{2f} \]

Figure 7-3 An elastic collision between two carts originally traveling toward each other

**Examples:**

1. Two metal spheres, suspended by vertical cords, initially touch each other. Sphere 1 with mass \( m_1 = 30 \text{ g} \) is pulled to the left to a height \( h_1 = 8.0 \text{ cm} \) and then released from rest. After swinging down, it undergoes an elastic collision with sphere 2 with mass \( m_2 = 75 \text{ g} \). What is the velocity of sphere 1, \( v_{1i} \), just before the collision? See Figure 7-4

![Figure 7-4 System of two masses for examples 1, 2, 3, 4, and 5](figure)

**Solution:**
From conservation of energy, we have:

\[
\frac{1}{2} m_1 v_{1i}^2 = m_1 g h_1
\]

\[
v_{1i} = \sqrt{2g h_1} = \sqrt{2 \times 9.8 \times 0.080} = 1.25 \text{ m/s}
\]

2. Two metal spheres, suspended by vertical cords, initially touch each other. Sphere 1 with mass \( m_1 = 30 \text{ g} \) is pulled to the left to a height \( h_1 = 8.0 \text{ cm} \) and then released from rest. After swinging down, it undergoes an elastic collision with sphere 2 with mass \( m_2 = 75 \text{ g} \). What is the velocity of sphere 1, \( v_{1f} \), just after the collision? See Figure 7-4

**Solution:**
Use Equation (5)

\[
v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_i = \frac{0.030 - 0.075}{0.030 + 0.075} \times 125 = -0.54 \text{ m/s}
\]

3. Two metal spheres, suspended by vertical cords, initially touch each other. Sphere 1 with mass \( m_1 = 30 \text{ g} \) is pulled to the left to a height \( h_1 = 8.0 \text{ cm} \) and then released from rest. After swinging...
down, it undergoes an elastic collision with sphere 2 with mass \( m_2 = 75 \text{ g} \). What is the velocity of sphere 2, \( v_{2f} \) just after the collision? See Figure 7.4

**Solution:**

Use Equation (6)

\[
v_{2f} = \frac{2m_1}{m_1 + m_2} v_y = \frac{2 \times 0.030}{0.030 + 0.075} \times 12.52 = 0.71 \text{ m/s}
\]

4. Two metal spheres, suspended by vertical cords, initially touch each other. Sphere 1 with mass \( m_1 = 30 \text{ g} \) is pulled to the left to a height \( h_1 = 8.0 \text{ cm} \) and then released from rest. After swinging down, it undergoes an elastic collision with sphere 2 with mass \( m_2 = 75 \text{ g} \). To what height, \( h_{1}' \) does the sphere 1 swing to the left after the collision? See Figure 7.4

**Solution:**

Use the conservation of energy here to get:

\[
h_1' = \frac{v_{1f}^2}{2g} = \frac{(-0.54)^2}{2 \times 9.8} = 0.015 \text{ m} = 1.5 \text{ cm}
\]

5. Two metal spheres, suspended by vertical cords, initially touch each other. Sphere 1 with mass \( m_1 = 30 \text{ g} \) is pulled to the left to a height \( h_1 = 8.0 \text{ cm} \) and then released from rest. After swinging down, it undergoes an elastic collision with sphere 2 with mass \( m_2 = 75 \text{ g} \). To what height, \( h_2 \) does the sphere 2 swing to the right after the collision? See Figure 7.4

**Solution:**

Use the conservation of energy to get:

\[
h_2 = \frac{v_{2f}^2}{2g} = \frac{(0.71)^2}{2 \times 9.8} = 0.026 \text{ m} = 2.6 \text{ cm}
\]

6. A ballistic pendulum is a device that was used to measure the speeds of bullets before electronic timing devices were invented. The device consists of a large block of wood of mass \( M = 5.4 \text{ kg} \), hanging from two long cords. A bullet of mass \( m = 9.5 \text{ g} \) is fired into the block, coming quickly to rest. The block + bullet then swing upward, their center of mass rising a vertical distance \( h = 6.3 \text{ cm} \) before the pendulum comes momentarily to rest at the end of its arc. What was the speed, \( v_{1i} \) of the bullet before it hit the block? See Figure 7.5

**Solution:**

Use the conservation of linear momentum:

\[
m_1 v_{1i} = (m_1 + m_2) V
\]

\[
v_{1i} = \frac{(m_1 + m_2) V}{m_1}
\]

Where \( m_1 = m = 9.5 \text{ g} \) and \( m_2 = M = 5.4 \text{ kg} \)

But the first step is to find \( V \), the velocity of the block + bullet just after impact. Since the mechanical energy (kinetic plus potential) is conserved after the collision, we have:
\[
\frac{1}{2} (m + M) V^2 = (m + M) gh
\]
\[
V = \sqrt{2gh} = \sqrt{2 \times 9.8 \times 0.063} = 1.11 \text{ m/s}
\]

Now we can find the speed of the bullet before it hit the block as follows
\[
v_{hi} = \frac{(m_1 + m_2) V}{m_1} = \frac{(0.0095 + 0.54)}{0.0095} \times 111 = 632 \text{ m/s}
\]

Figure 7-5 The ballistic pendulum

7. What is the linear momentum of an automobile with weight 1000 kg traveling at 60 km/hr?

Solution:
\[
p = mv = 1000 \times 60 \times 1000 / 3600 = 1.66 \times 10^3 \text{ kg.m/s}
\]

8. Suppose that your mass is 80 kg. How fast would you have to run to have the same momentum as a 1600 kg car moving at 1.2 km/hr?

Solution:
\[
v = \frac{m_2 v_2}{m_1} = \frac{1600 \times 1.2}{80} = 24 \text{ km/hr}
\]

9. Two carts, with masses \(m_1 = 1.6 \text{ kg}\) and \(m_2 = 2.4 \text{ kg}\) travel on a frictionless surface and collide. What is the velocity of cart 1 after the collision, knowing that \(v_{1i} = 5.5 \text{ m/s}\), \(v_{2i} = 2.5 \text{ m/s}\), and \(v_{2f} = 4.9 \text{ m/s}\)? See Figure 7-6

Figure 7-6 The cart with mass \(m_1\) collides with cart with mass \(m_2\)

Solution:
Since momentum is conserved we have:
\[ m_1v_{1i} + m_2v_{2i} = m_1v_{1f} + m_2v_{2f} \]
\[ v_{1f} = \frac{m_1v_{1i} + m_2v_{2i} - m_2v_{2f}}{m_1} \]
\[ = \frac{1.6 \times 5.5 + 2.4 \times 2.5 - 2.4 \times 4.9}{1.6} = 1.9 \text{ m/s} \]

10. Is the collision of carts in Figure 7-6 elastic?

**Solution:**
To answer this we calculate the kinetic energy of the system before and after the collision as follows:

Before: \[ \frac{1}{2} m_1v_{1i}^2 + \frac{1}{2} m_2v_{2i}^2 = \frac{1}{2} \times 16 \times 5.5^2 + \frac{1}{2} \times 2.4 \times 2.5^2 = 31.7 \text{ J} \]

After: \[ \frac{1}{2} m_1v_{1f}^2 + \frac{1}{2} m_2v_{2f}^2 = \frac{1}{2} \times 1.6 \times 1.9^2 + \frac{1}{2} \times 2.4 \times 4.9^2 = 31.7 \text{ J} \]

Yes, the collision is elastic as the kinetic energies before and after the collision are the same

**Equipment and Materials**
- Force sensor
- Motion sensor
- Two carts
- Frictionless track
- Computer system with DataStudio™

**Procedure**

**Inelastic Collisions**
1. Adjust the frictionless track so that it is horizontal
2. Place the motion sensor at one end of the track
3. Find the masses of the two carts and write them down into your laboratory report
4. Place the carts on the track so that the Velcro pieces face each other
5. Set up the motion sensor so that it can measure the speed of the first cart as it moves toward and collides with the second cart which is at rest
6. Select a graph and choose to plot the velocity of the cart as a function of time
7. Start the program and give the first cart a slight push. Stop the program after the two carts reach the far end of the track
8. Use the **Smart Tool**, and from this graph find the velocity of cart 1 before collision, \( v_{1i} \) and the velocity, \( V \) of the carts after collision
9. Calculate the velocity of the carts after the collision from the theory, using the value obtained above for the velocity of cart 1 before the collision
10. Calculate the % error
11. Repeat the same for a different initial cart 1 speed
12. Attach your graphs to your lab report

**Elastic Collisions**
1. Level the frictionless track so that it is horizontal
2. Attach two motion sensors on either end of the track
3. Measure the masses of the two carts
4. Adjust each sensor so it can measure the motion of the cart as it moves from the end of the track to the middle and back again
5. Choose to plot the velocity of the carts as a function of time for both sensors
6. Place the carts on the track so that the magnets face each other
7. Position each cart about 15 cm from the sensors
8. Click **Start** and push each cart gently so they collide
9. Stop the program after the collision and when the carts are back at the ends of the track
10. Use the Smart Tool to find the change in velocities of the carts before the collision to just after the collision
11. Attach the graph to the lab report
Experiment 7 Laboratory Report

Linear momentum

Section _______ Laboratory bench number _______

Date: ______________________________________

Students:
________________________________________________________________________
________________________________________________________________________
________________________________________________________________________
________________________________________________________________________

Inelastic Collisions

3. Write down the masses of the two carts.

\[ m_1 = \quad m_2 = \]

8. 9. Use the Smart Tool, and from this graph find the velocity of cart 1 before collision, \( v_{1i} \), and the velocity, \( V \), of the carts after collision

10. Calculate the velocity of the carts after the collision from the theory, using the value obtained above for the velocity of cart 1 before the collision

11. Calculate the % error.

12. Repeat the same for a different initial cart 1 speed

13. Attach your graphs to your lab report

<table>
<thead>
<tr>
<th>Trial #</th>
<th>( v_{1i} ) (m/s) From graph</th>
<th>( V ) (m/s) From graph</th>
<th>( V ) (m/s) From theory</th>
<th>% Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Elastic Collisions

3. Write down the masses of the two carts

\[ m_1 = \quad m_2 = \]

10. Use the Smart Tool to find the change in velocities of the carts before the collision to just after the collision

11. Attach the graph to your laboratory report

12. Write down your results in the table below
Questions

1. A cart with mass 340 g moving on a frictionless track at an initial speed of 1.2 m/s undergoes an elastic collision with an initially stationary cart of unknown mass. After the collision, the first cart continues in its original direction at 0.66 m/s.
   
   (a) What is the mass of the second cart?
   (b) What is its speed after impact?

2. Derive equations (5) and (6) starting from Equations (3) and (4). Show all your steps
Experiment 7 Questions

Conservation of Linear Momentum - Collisions

This questionnaire has some typical questions on experiment 7. All students who are taking the laboratory course of University Physics I must be able to correctly answer it before trying to make the experiment.

NOTE: Questions 1 to 5 refer to Figure 7-7 below. Assume that the acceleration of gravity is $g = 9.8 \text{ m/s}^2$, and the collision is elastic.

Figure 7-7 This system of two spheres is for questions 1 to 5

1. Two metal spheres, suspended by vertical cords, initially touch each other. See Figure 7-7 above. Sphere 1 with mass $m_1 = 40 \text{ g}$ is pulled to the left to a height $h_1 = 10.0 \text{ cm}$ and then released from rest. After swinging down, it undergoes an elastic collision with sphere 2 with mass $m_2 = 60 \text{ g}$ which is at rest. What is the velocity, $v_{1i}$, of sphere 1 just before the collision?
   
   a. $1.61 \text{ m/s}$
   b. $14.0 \text{ m/s}$
   c. $1.96 \text{ m/s}$
   d. $1.40 \text{ m/s}$
   e. $3.84 \text{ m/s}$

2. Two metal spheres, suspended by vertical cords, initially touch each other. See Figure 7-7 above. Sphere 1 with mass $m_1 = 40 \text{ g}$ is pulled to the left to a height $h_1 = 10.0 \text{ cm}$ and then released from rest. After swinging down, it undergoes an elastic collision with sphere 2 with mass $m_2 = 60 \text{ g}$ which is at rest. What is the velocity, $v_{1f}$, of sphere 1 just after the collision?

   a. $-0.77 \text{ m/s}$
   b. $-0.28 \text{ m/s}$
   c. $-2.8 \text{ m/s}$
   d. $-0.39 \text{ m/s}$
   e. $-0.32 \text{ m/s}$

3. Two metal spheres, suspended by vertical cords, initially touch each other. See Figure 7-7 above. Sphere 1 with mass $m_1 = 40 \text{ g}$ is pulled to the left to a height $h_1 = 10.0 \text{ cm}$ and then released from rest. After swinging down, it undergoes an elastic collision with sphere 2 with mass $m_2 = 60 \text{ g}$ which is at rest. What is the velocity, $v_{2f}$, of sphere 2 just after the collision?

   a. $1.12 \text{ m/s}$
   b. $1.29 \text{ m/s}$
   c. $18.0 \text{ m/s}$
   d. $1.57 \text{ m/s}$
   e. $3.07 \text{ m/s}$
4. Two metal spheres, suspended by vertical cords, initially touch each other. See Figure 7-7 above. Sphere 1 with mass $m_1 = 40$ g is pulled to the left to a height $h_1 = 10.0$ cm and then released from rest. After swinging down, it undergoes an elastic collision with sphere 2 with mass $m_2 = 60$ g which is at rest. To what height $h_1'$ does the sphere 1 swing to the left after the collision?
   a. 0.01 cm
   b. 40 cm
   c. 0.40 cm
   d. 2.0 cm
   e. We do not have enough data

5. Two metal spheres, suspended by vertical cords, initially touch each other. See Figure 7-7 above. Sphere 1 with mass $m_1 = 40$ g is pulled to the left to a height $h_1 = 10.0$ cm and then released from rest. After swinging down, it undergoes an elastic collision with sphere 2 with mass $m_2 = 60$ g which is at rest. To what height $h_2$ does the sphere 2 swing to the right after the collision?
   a. 6.40 cm
   b. 16.5 cm
   c. 8.49 cm
   d. It goes up to the original height of mass 1 since the energy is conserved
   e. It does not move since its mass is larger than that of the other sphere

NOTE: Questions 6 and 7 refer to Figure 7-8 below. Assume that the acceleration of gravity is $g = 9.8$ m/s$^2$, and the collision is totally inelastic.

6. A ballistic pendulum is a device that was used to measure the speeds of bullets before electronic timing devices were invented. The device consists of a large block of wood of mass $M = 6.0$ kg, hanging from two long cords. See Figure 7-8 above. A bullet of mass $m = 30$ g is fired into the block, coming quickly to rest. The block + bullet then swing upward, their center of mass rising a vertical distance $h = 8.4$ cm before the pendulum comes momentarily to rest at the end of its arc. What was the speed $V$ of the block + bullet just after impact?
   a. 12.8 m/s
   b. The block is not going to move, its velocity will be zero
   c. 1.65 m/s
   d. 1.28 m/s
   e. 164 m/s
7. A ballistic pendulum is a device that was used to measure the speeds of bullets before electronic timing devices were invented. The device consists of a large block of wood of mass \( M = 6.0 \) kg, hanging from two long cords. See Figure 7-8 above. A bullet of mass \( m = 30 \) g is fired into the block, coming quickly to rest. The block + bullet then swing upward, their center of mass rising a vertical distance \( h = 8.4 \) cm before the pendulum comes momentarily to rest at the end of its arc. What was the speed of the bullet, \( v_i \) before it hit the block?
   a. 2.57 km/s
   b. Not enough data
   c. 332 m/s
   d. 257 m/s
   e. \( 33 \times 10^3 \) km/s

8. What is the linear momentum of an automobile with a mass of 1500 kg traveling at 72 km/hr?
   a. \( 1.08 \times 10^2 \) kg m/s
   b. What we need is the automobile’s weight
   c. \( 30 \times 10^3 \) kg m/s
   d. \( 1.0 \times 10^5 \) kg m/s
   e. \( 36 \times 10^3 \)

9. Suppose that your mass is 60 kg. How fast would you have to run to have the same momentum as a 1500 kg car moving at 12 km/hr?
   a. 0.48 km/hr
   b. 130 km/hr
   c. 60 km/hr
   d. 1500 km/hr
   e. 25 km/hr

NOTE: Questions 10 and 11 refer to Figure 7-9 below. Assume that \( m_1 = 1.2 \) kg, \( v_{1i} = 4.0 \) m/s, \( m_2 = 1.8 \) kg, \( v_{2i} = 3.0 \) m/s, and that the collision is elastic

![Figure 7-9 The cart with mass \( m_1 \) collides elastically with cart with mass \( m_2 \)](image)

10. Two carts travel on a horizontal frictionless surface and collide elastically as seen above in Figure 9. What is the velocity of cart 1, \( v_{1f} \) after the collision?
    a. 2.1 m/s
    b. 1.9 m/s
    c. 7.4 m/s
    d. \(-3.1 \) m/s
    e. 3.4 m/s

11. Two carts travel on a horizontal frictionless surface and collide elastically as seen above in Figure 9. What is the velocity of cart 2, \( v_{2f} \) after the collision?
    a. \(-2.4 \) m/s
    b. 1.8 m/s
    c. 3.2 m/s
    d. 2.9 m/s
    e. 3.8 m/s